CS 421 Lecture 16

- Functional programming
- Higher-order functions

History of functional languages

- LISP, APL (1960)
- ML (1976) – Milner, "A theory of type polymorphism in programming"
- SASL (1976) – lazy evaluation
- SCHEME (1975) – Guy Steele – dialect of LISP with higher-order functions ("lexical scope")
- Standard ML, CAML (1980's)
- Erlang (1987) – Ericsson
- Haskell (1990) – lazy evaluation
- Python, ...

Functional languages

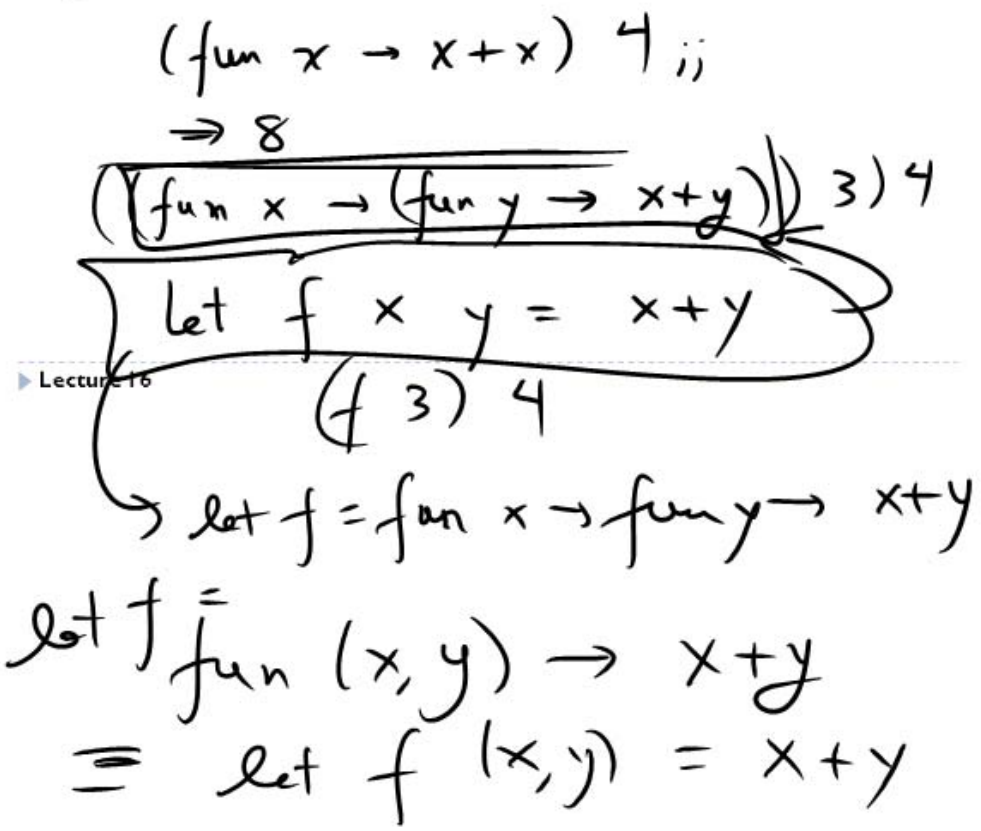
- Expressions (rather than statements)
  - Absence of side effects
  - "Large values"
- Dynamic memory allocation
- Recursion
- Static type checking with polymorphic types (ML, Haskell)
- **Higher-order functions**, aka "functions as values" (Scheme, ML, Haskell, Python, ...)
- Lazy evaluation (Haskell)

Higher-order functions

- Functions are a type of value ("first-class functions")
  - Define anonymously
  - Pass as arguments
  - Bind to names
  - Assign to variables
  - Return from functions

Anonymous functions in Ocaml

- Notation: "fun x -> e" – Ocaml expression whose values is a function.
- "let f = fun x -> e" is equivalent to "let f x = e"



Passing functions as arguments

Higher-order functions in List module:

```
map : (α->β)->α list -> β list
```

applies a function to each element of a list –

```
map f [x1;x2;...xn] = [f x1;f x2;...f xn]
```

E.g.

```
let lis = [1;2;3;4]
let incr x = x + 1
map incr lis => [2;3;4;5]
```

= let incr = fun x → x+1

or equivalently

```
map (fun x -> x + 1) lis
```

Passing functions as arguments

```
fold_right f [x1;x2;...xn] z
    = f x1 (f x2 (...(f xn z)...))
fold_right : (α->β->β)->(α list)->β->β
```

(+)

```
fold_right (fun x y -> x+y) lis 0 => 10
```

[1;2;3;4]

(+) 1 (fr (+) [2;3;4] 0)

= ... = (+) 1 ((+) 2 ((+) 3 ((+) 4 0)))

(Note: can use "(+)" for function argument.)

Passing functions as arguments

```
fold_right (fun x -> fun y -> x :: y) lis []
  => lis
fold_right (fun x -> fun y -> x :: y) lis lis
  => lis @ lis
fold_right
 (fun x -> fun y -> (x + (hd y))::y) lis [0]
  [1;2;3;4]  => [10;9;7;4;0]
fold_right
  (fun x -> fun (y::ys) -> (x + y)::ys) lis [0]
```

map f lis = fold_right (fun x y -> f x :: y) lis []

map is a special case of fold_right

Passing functions as arguments

- Define `f`, `z` such that `fold_right f lis z` = the pair of lists `(l1,l2)` where `l1` contains the elements of `lis` that are < 0, and `l2` contains the rest

```
f = fun x ->
      fun (l1,l2) ->
        if x < 0
        then (x::l1,l2)
        else (l1,x::l2)
```

Passing functions as arguments

```
fold_left : (α->β->α)->α->β list->α
fold_left f [x1;x2;...xn] x
 = f f(...(f z x1) x2)...) xn

fold_left (+) lis 0 => sum of lis
```

Define mapplesauce $[x_1; x_2; \ldots] = [x_1+1; x_2+1; \ldots]$

```
let rec mapplesauce lis =
    if lis = [] then []
      else (hd lis)+1 :: mapplesauce (tl lis)

let rec map f lis =
    if lis = [] = []
      else f (hd lis) :: map f (tl lis)
```

Defining higher-order functions

```
let rec map f lis =
  if lis = [] then []
  else f (hd lis)::map f (tl lis)

let rec fold_right f lis z =
  if lis = [] then z
  else f (hd lis)
    (fold_right f (tl lis) z)
```

map incr lis

map (fun x → x * x) lis

map : $(\alpha \to \beta) \to \alpha$ list $\to \beta$ list

let mapincr = map incr;;

int list → int list

mapincr [ ]

map incr [ ]

Understanding higher-order functions

Two approaches: Substitution, or environment/closure model

Consider: let add1 = map (fun x -> x+1)

Returns: fun lis -> if lis = [] then []
else f (hd lis)::map f (tl lis)

But this has "f" as a free variable.

Question: when add1 is applied, where does the value of f come from?

Substitution model

Replace free variable with its value:

```
    map (fun x -> x+1)
 = fun lis -> if lis = [] then []
        else (fun x -> x+1) (hd lis)::map (fun x -> x+1) (tl lis)
```

(Note: no free variables any more.)

Environment/closure model

Put free variables in a data structure called an *environment*:

{f $\rightarrow$ fun x -> x+1}

Keep expression and environment together in a pair:

(fun lis -> if lis = [] then []
else f (hd lis)::map f (tl lis), {f $\rightarrow$ fun x -> x+1})

This pair is called a *closure*.

After applying map to the function, the value is always kept in the form of the closure, never as just the expression.

Evaluating expressions in OCaml

Two approaches: substitution model
environment model

Substitution

Eval. constant $n \Rightarrow n$
constant $[n_1; n_2; n_3] \Rightarrow [n_1; n_2; n_3]$

$e_1 + e_2$: (1) $e_1 \Rightarrow n_1$
(2) $e_2 \Rightarrow n_2$
(3) $\Rightarrow n_1 + n_2$

if e_1 then e_2 else e_3 : (1) $e_1 \Rightarrow t$
(2) if t = true, eval $e_2 \Rightarrow v$
o.w. eval $e_3 \Rightarrow v$
(3) $\Rightarrow v$

let $f = e_1$ in e_2 : $\equiv$ (fun $f \rightarrow e_2$) e_1

fun $x \rightarrow e$: return fun $x \rightarrow e$

$e_1\ e_2$: (1) $e_1 \Rightarrow$ fun $x \rightarrow e$
(2) $e_2 \Rightarrow v$
(3) subst. v for x in $e \Rightarrow v'$